On considère la suite (u_n) définie sur $\mathbb{N}^*$ par $u_n = \int_{-1}^{0} \frac{(x+1)^n}{\sqrt{x^2+2x+2}} dx$

3 a Montrer que pour tout $n \in \mathbb{N}^*$, $u_{n+2} = \frac{\sqrt{2}}{n+2} - \frac{n+1}{n+2} \times u_n$.

$$u_{n+2} = \int_{-1}^{0} \frac{(x+1)^{n+2}}{\sqrt{x^2+2x+2}} dx$$

on pose

$u(x) = (x+1)^{n+1} \longrightarrow u'(x) = (n+1)(x+1)^n$

$v'(x) = \frac{(x+1)}{\sqrt{x^2+2x+2}} \longrightarrow v(x) = \sqrt{x^2+2x+2}$

$$u_{n+2} = \left[(x+1)^{n+1} \cdot \sqrt{x^2+2x+2}\right]_{-1}^{0} - (n+1) \int_{-1}^{0} (x+1)^n \cdot \sqrt{x^2+2x+2}\, dx$$

$$= \sqrt{2} - (n+1) \int_{-1}^{0} (x+1)^n \cdot \sqrt{x^2+2x+2}\, dx$$

$$= \sqrt{2} - (n+1) \int_{-1}^{0} (x+1)^n \cdot \frac{x^2+2x+2}{\sqrt{x^2+2x+2}}\, dx \qquad (x^2+2x+1)+1$$

$$= \sqrt{2} - (n+1) \int_{-1}^{0} (x+1)^n \cdot \frac{(x+1)^2+1}{\sqrt{x^2+2x+2}}\, dx$$